Longitudinal and Transverse Current Distributions on Coupled Microstrip Lines

MASANORI KOBAYASHI, MEMBER, IEEE, AND HIDETOSHI MOMOI

Abstract —The normalized longitudinal and transverse current distributions on coupled microstrip lines are obtained for even and odd modes by using the charge conservation formula and the charge distributions calculated by a Green's function technique. Their dependence on the shape ratios w/h and s/h and on the relative permittivity ϵ^* of the substrate is shown.

I. Introduction

IN CALCULATING accurately the dispersion characteristics of microstrip line, the spectral-domain analysis proposed by Itoh and Mittra [1], [2] has powerful features. In this analysis, the choice of the basis functions is important for numerical efficiency. If the first few basis functions approximate the actual unknown current distributions reasonably well, the necessary size of the matrix can be held small for a given accuracy of the solution, so that CPU time can be saved.

Using the spectral-domain analysis, a previous paper [9] confirmed that an unsuitable choice of basis functions caused significant discrepancies between many computed results, as shown by Kuester and Chang [3, Fig. 2]. The closed-form expressions for the normalized current distributions [8] used then were obtained by approximating the results obtained using the method derived by Denlinger [4]. In addition, the results shown in the previous paper [9] have a high degree of accuracy for use as a "standard" of an effective relative permittivity. This means not only that the spectral-domain analysis has powerful features but also that the method derived by Denlinger [4] is useful for obtaining the normalized current distributions.

The present work determines the normalized longitudinal and transverse current distributions for even and odd modes on "coupled" microstrip line, shown in Fig. 1, with concern for numerical efficiency in obtaining the dispersion characteristics. However, the literature determining current distributions is sparse even for the single microstrip line, as indicated in a previous paper [8]. For an example of the current distributions on coupled microstrip line, we may cite the paper by Krage and Haddad [5].

Manuscript received May 23, 1987; revised October 13, 1987.
The authors are with the Department of Electrical Engineering, Faculty of Engineering, Ibaraki University, Hitachi, Ibaraki, Japan.
IEEE Log Number 8718867.

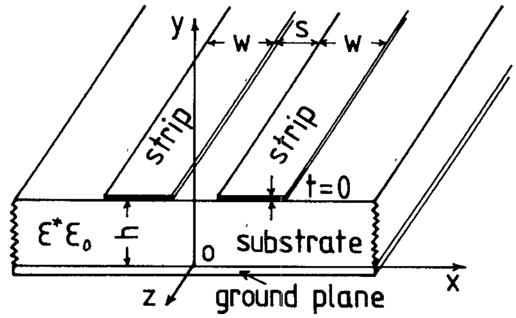

Fig. 1. Configuration of coupled microstrip line.

In this paper, the charge distributions for even and odd modes are calculated with a high degree of accuracy using a Green's function technique [7] for various cases with $\epsilon^* = 1$, 2, 4, 8, 16, and $0.1 \leqslant w/h \leqslant 10$, and $0.1 \leqslant s/h \leqslant 10$, respectively, and are used to calculate the current distributions.

II. Procedure for Calculating the Longitudinal and Transverse Current Distributions

The coupled microstrip line considered in this article is shown in Fig. 1, where two infinitesimally thin strips and the ground plane are taken as perfect conductors. The structure is divided into two regions, corresponding to the air and dielectric substrate regions of the structure. It is also assumed that the substrate material is lossless and its relative permittivity and permeability are ϵ^* and $\mu^*(=1)$, respectively.

The quasi-TEM characteristics of this coupled microstrip line can be obtained by a Green's function technique [7] if a suitable Green's function can be derived. Therefore, only the appropriate Green's function is described and also the expression for the desired unknown charge distribution.

Consider only the region $x \geqslant 0$ (due to symmetry) as that for the boundary-value problem. This problem is two-dimensional, and the value of the Green's function at the point (x, h) for unit source charge at the point (x_0, h)

0018-9480/88/0300-0588$01.00

on the strip is determined as follows:

$$G(x,h;x_0,h)=\frac{1}{2\pi\epsilon_0(1+\epsilon^*)}\sum_{n=1}^{\infty}K^{n-1}\cdot\ln\left[\left\{\frac{4n^2+\left(\frac{x-x_0}{h}\right)^2}{4(n-1)^2+\left(\frac{x-x_0}{h}\right)^2}\right\}\cdot\left\{\frac{4n^2+\left(\frac{x+x_0}{h}\right)^2}{4(n-1)^2+\left(\frac{x+x_0}{h}\right)^2}\right\}^g\right] \tag{1}$$

where ϵ_0 denotes the permittivity of vacuum, $K=(1-\epsilon^*)/(1+\epsilon^*)$, and $g=1$ for the even mode and -1 for the odd mode.

Also, we express the desired unknown charge distribution $\sigma(x_0,h)$ on the strip as follows:

$$\sigma(x_0,h)=\sigma_j+(\sigma_{j+1}-\sigma_j)\frac{x_0-x_j}{x_{j+1}-x_j} \tag{2}$$

where we take

$$x_j=\begin{cases}\frac{s+w}{2}-\frac{w}{2}\left\{1-\left(1-\frac{q+1-j}{q}\right)^3\right\}, & j=1,2,\cdots,q(q=m/2)\\ \frac{s+w}{2}+\frac{w}{2}\left\{1-\left(1+\frac{q+1-j}{q}\right)^3\right\}, & j=q+1,q+2,\cdots,m+1\end{cases} \tag{3}$$

in order to make $\sigma(x_0,h)$ approach the true charge distribution.

In this paper, we took $m=40$ in (3) and the truncated number $n=N$ in the infinite series (1) as satisfying

$$|K|^{N-1}<10^{-6}. \tag{4}$$

The high degree of accuracy of the charge distributions calculated here was checked from the results with larger m. It may also be supported by the fact that the coupled microstrip line becomes a single microstrip line with w/h when $s/h\to\infty$ and with $2w/h$ when $s/h\to 0$. In [7] it was shown in detail that the Green's function technique has a high degree of accuracy for a single microstrip line.

Let the charge distributions for the even mode be denoted by $\sigma_0^E(x)$ for $\epsilon^*=1$ and σ^E for $\epsilon^*\neq 1$. Let the charge distributions for the odd mode be denoted by $\sigma_0^O(x)$ for $\epsilon^*=1$ and σ^O for $\epsilon^*\neq 1$.

Now the continuity equation between the longitudinal and transverse current distributions, i_z^P and i_x^P, and the charge distribution $\sigma^P(x)$ is expressed as

$$\frac{\partial i_x^P}{\partial x}+\frac{\partial i_z^P}{\partial z}=-j\omega\sigma^P(x)e^{-j\beta^P(f)z} \tag{5}$$

where an $e^{j\omega t}$ time variation is assumed, and the superscript P becomes E or O for even or odd modes, respectively. $\beta^P(f)$ denotes the phase constant $(=\omega/v^P(f)$, $\omega=2\pi f)$, $v^P(f)(=c/\sqrt{\epsilon_{\text{eff}}^{*P}(f)})$ the phase velocity, $\epsilon_{\text{eff}}^{*P}(f)$ the effective relative permittivity at the frequency f, and c the velocity of light in free space.

Let the total longitudinal current be denoted by $I\exp(-j\beta^P(f)z)$, and denote the quantity $I/v^P(f)$ by Q. Using the Green's function technique, we can obtain the charge distribution $\sigma_0^P(x)$ on a strip of the coupled microstrip line without the substrate $(\epsilon^*=1)$ for a given total charge per unit length $Q/\epsilon_{\text{eff}}^{*P}(0)$. Let $i_z^P(x)$ be denoted by the following approximate relation:

$$i_z^P(x)=\epsilon_{\text{eff}}^{*P}(0)\sigma_0^P(x)v^P(f)e^{-j\beta^P(f)z}. \tag{6}$$

Substituting (6) into (5), we can derive the following approximate expression for obtaining the transverse current distribution on the strip:

$$i_x^P(x)=-j\omega(\operatorname{sgn}x)\cdot\int_0^x\left\{\sigma^P(x)-\epsilon_{\text{eff}}^{*P}(0)\sigma_0^P(x)\right\}dx\,e^{-j\beta^P(f)z} \tag{7}$$

where

$$\operatorname{sgn}x=\begin{cases}-1, & x<0\\ +1, & x>0.\end{cases}$$

These expressions for $i_z^P(x)$ and $i_x^P(x)$ are an extension of those derived by Denlinger [4] for the single microstrip line.

III. Results of Normalized Longitudinal Current Distributions

Normalizing the longitudinal current distribution in (6) to its value at the midpoint (W_s,h) on the strip, we obtain

$$i_z^P(x)/i_z^P(W_s)=\sigma_0^P(x)/\sigma_0^P(W_s) \tag{8}$$

where $W_s=s/2+w/2$. Therefore, the normalized longitudinal current distribution $i_z^P(x)/i_z^P(W_s)$ can be expressed approximately by the normalized charge distribution $\sigma_0^P(x)/\sigma_0^P(W_s)$ for the case without the substrate.

Fig. 2 shows the normalized charge distributions $\sigma_0^E(x)/\sigma_0^E(W_s)$ on the strip for the even mode when $w/h=1$ and hence also the normalized longitudinal current distributions $i_z^E(x)/i_z^E(W_s)$. In Fig. 2, the curve for $s/h=0$ denotes the result for the single microstrip line. This curve becomes the lower bound of curves for the coupled microstrip line on the left half of the strip and the upper bound on the right half. On the other hand we can easily understand that the curve for the single microstrip line with $w/h=1(s/h\to\infty)$ becomes the upper bound on the left half and the lower bound on the right half, although its curve is not shown here. We may also derive the closed-form expression for $i_z^E(x)/i_z^E(W_s)$ [10], but it is not shown here since it is complicated. The result obtained is shown by the dashed line for the case of $w/h=1$ and $s/h=1$ in Fig. 2.

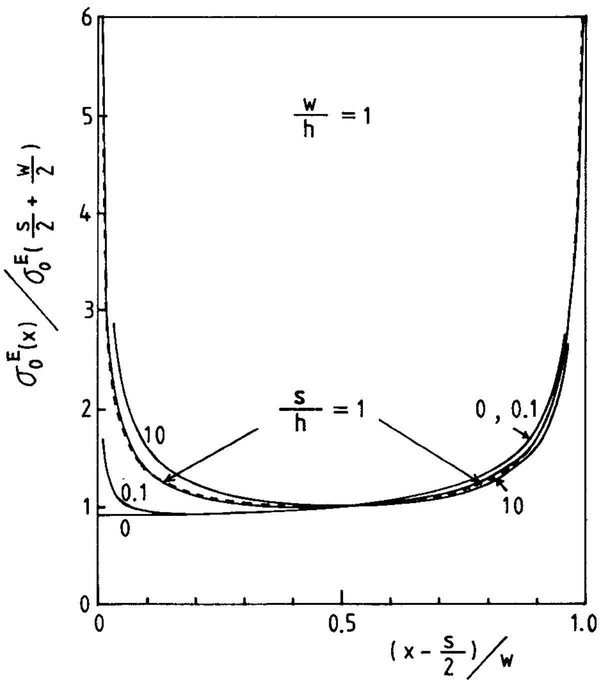

Fig. 2. Normalized charge distributions $\sigma_0^E(x)/\sigma_0^E(W_s)(=i_z^E(x)/i_z^E(W_s))$ on the strip for the cases of $w/h=1$ (even mode). $W_s=s/2+w/2$. —— Present method. ---- Approximate formula [10].

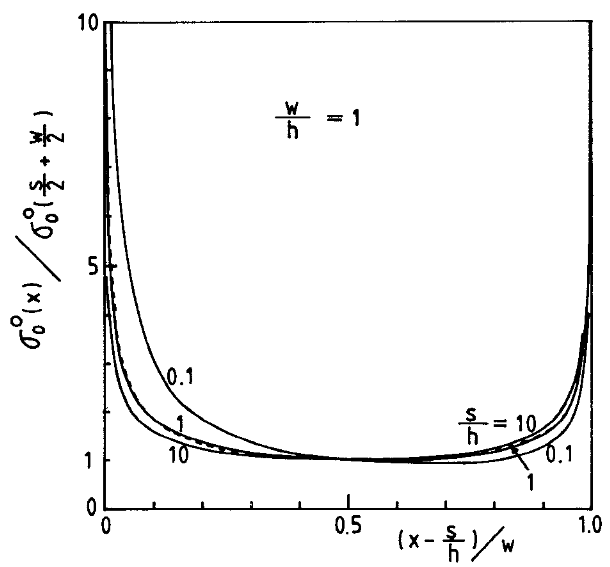

Fig. 3. Normalized charge distributions $\sigma_0^O(x)/\sigma_0^O(W_s)(=i_z^O(x)/i_z^O(W_s))$ on the strip for the cases of $w/h=1$ (odd mode). $W_s=s/2+w/2$. —— Present method. ---- Approximate formula [10].

Fig. 3 shows the normalized charge distributions $\sigma_0^O(x)/\sigma_0^O(W_s)$ on the strip for the odd mode when $w/h=1$, and consequently the normalized longitudinal current distributions $i_z^O(x)/i_z^O(W_s)$. The dashed line for the case of $w/h=1$ and $s/h=1$ in Fig. 3 shows the result obtained by a closed-form expression for $i_z^O(x)/i_z^O(W_s)$ [10].

Comparing the curves shown in Figs. 2 and 3, the current distribution tends to concentrate at the outer edge $(x=s/2+w)$ of the strip for the even mode, but at the inner edge $(x=s/2)$ of strip for the odd mode. The curves

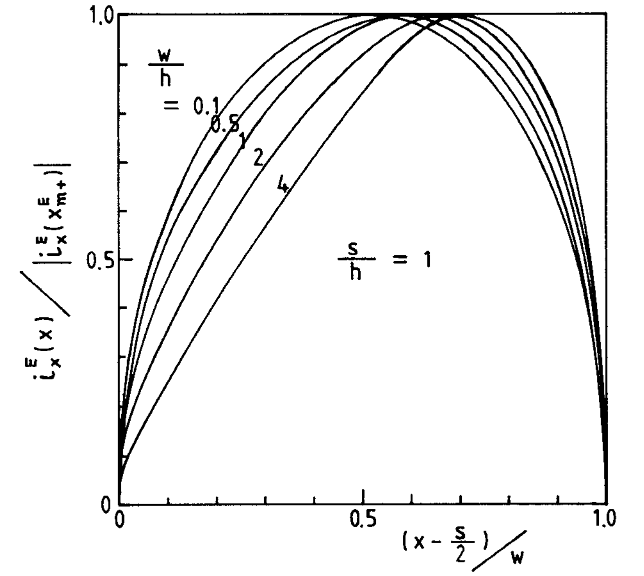

Fig. 4. Normalized transverse current distributions $i_x^E(x)/|i_x^E(x_{m+}^E)|$ on the strip for the cases of $s/h=1$, $w/h=0.1, 0.5, 1, 2, 4$, and $\epsilon^*=2, 4, 8, 16$ (even mode). Note: the curves for the various ϵ^* coincide.

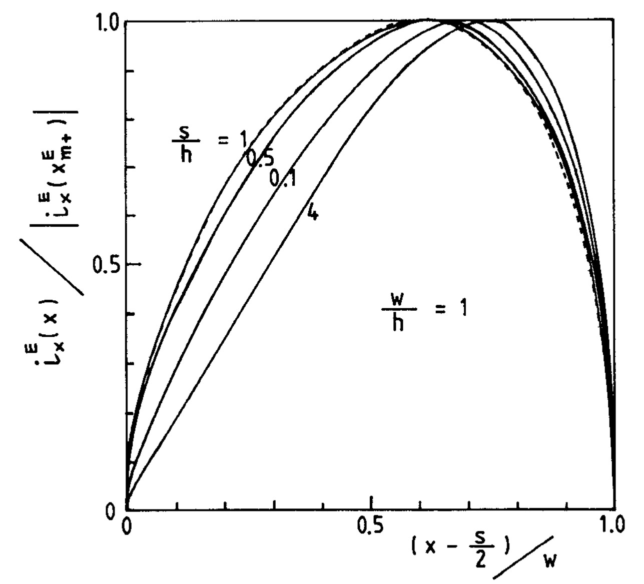

Fig. 5. Normalized transverse current distributions $i_x^E(x)/|i_x^E(x_{x+}^E)|$ on the strip for the cases of $w/h=1$, $s/h=0.1, 0.5, 1, 4$, and $\epsilon^*=2, 4, 8, 16$ (even mode). —— Present method. ----- Approximate formula [10].

satisfy the edge singularity [6] which requires that they approach the inner edge and the outer edge of a strip with the singularities $|x-s/2|^{-1/2}$ and $|x-(s/2+w)|^{-1/2}$, respectively.

IV. Results for Normalized Transverse Current Distributions

The current distribution $i_x^P(x)/|i_x^P(x_m)|$ normalized to $|i_x^P(x_m)|$ at the extremum point $x=x_m$ can be calculated by substituting the charge distributions $\sigma_0^P(x)$ and $\sigma^P(x)$ and the effective relative permittivity $\epsilon_{\text{eff}}^{*P}(0)$ at the frequency $f=0$ into (7).

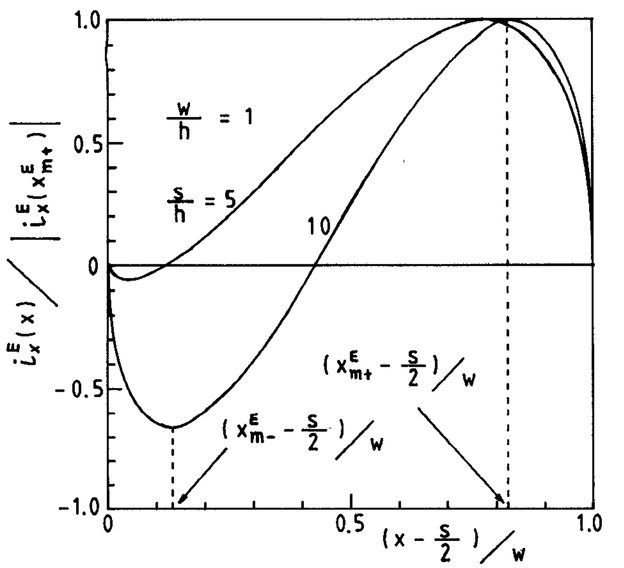

Fig. 6. Normalized transverse current distributions $i_x^E(x)/|i_x^E(x_{m+}^E)|$ on the strip for the cases of $w/h=1$, $s/h=5$, 10 and $\epsilon^*=2$, 4, 8, 16 (even mode).

Fig. 4 shows the results of the transverse current distribution $i_x^E(x)/|i_x^E(x_{m+}^E)|$ on the strip normalized to $|i_x^E(x_{m+}^E)|$ at the positive extremum point $x=x_{m+}^E$ when $s/h=1$. Fig. 5 shows similarly the results when $w/h=1$. On the other hand, we obtain interesting results for the cases $w/h=1$ and $s/h=5$, 10, as shown in Fig. 6. The curves have both positive and negative extrema. However, for single microstrip line the curve has only a positive value [8, Figs. 8 and 9]. Fig. 7 shows the positions of positive and negative extrema on the curves of the transverse current distributions for the even mode, and the negative to positive extrema ratios. The curves are obtained by taking the arithmetic mean of the results for the cases of $\epsilon^*=2$, 4, 8, 16. We can derive a closed-form expression for $i_x^E(x)/|i_x^E(x_{m+}^E)|$ to approximate the calculated results for the cases with only a positive extremum [10], but again it is not given here since it is complicated. The result obtained is shown by the dashed line for the case $w/h=1$ and $s/h=1$ in Fig. 5.

Fig. 8 shows the results of the transverse current distributions $i_x^O(x)/|i_x^O(x_{m-}^O)|$ on the strip normalized to $|i_x^O(x_{m-}^O)|$ at the negative extremum point $x=x_{m-}^O$ when $w/h=1$. Fig. 9 shows the results of the normalized transverse current distributions on the strip when $w/h=1$. We can derive the closed-form expression for $i_x^O(x)/|i_x^O(x_{m-}^O)|$ to approximate the results for the cases with only a negative extremum [10]. The result obtained is shown by the dashed line for the case of $w/h=1$ and $s/h=1$ in Fig. 9. Fig. 10 shows the positions of negative and positive extrema on the curves of the transverse current distributions for the odd mode, and the positive to negative extrema ratios.

Comparing the curves shown in Figs. 6 and 9, we notice that the larger extremum of the normalized transverse current distribution takes a positive value for the even mode but a negative value for the odd mode. For the even

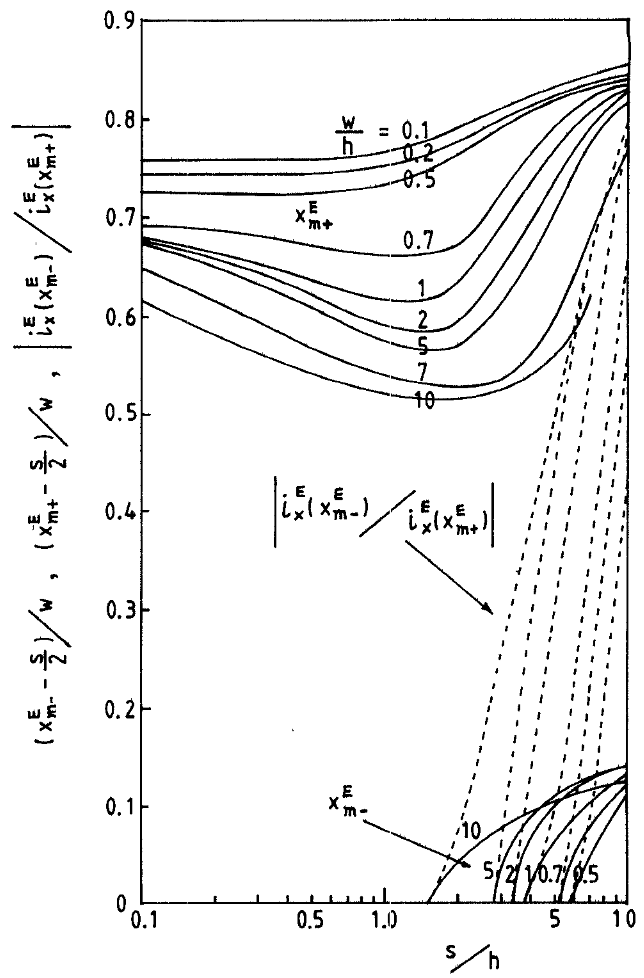

Fig. 7. The results for the positive and negative extrema of transverse current distributions (even mode).

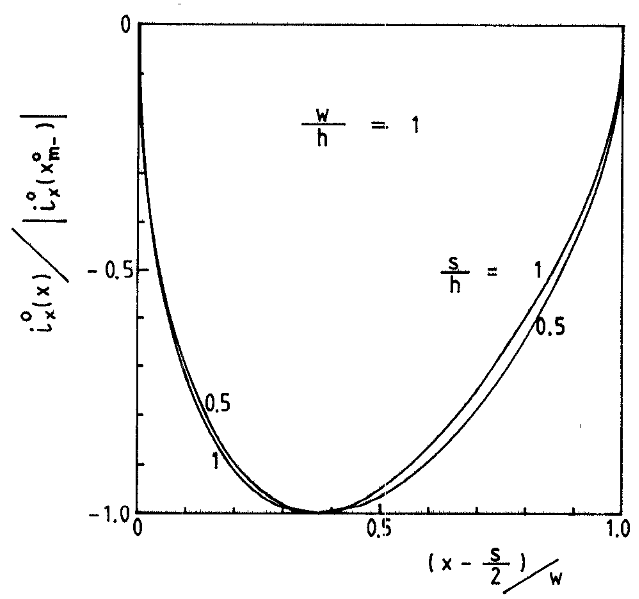

Fig. 8. Normalized transverse current distributions $i_x^O(x)/|i_x^O(x_{m-}^O)|$ on the strip for the cases of $w/h=1$, $s/h=0.5$, 1, and $\epsilon^*=2$, 4, 8, 16 (odd mode) The curves for all ϵ^* coincide.

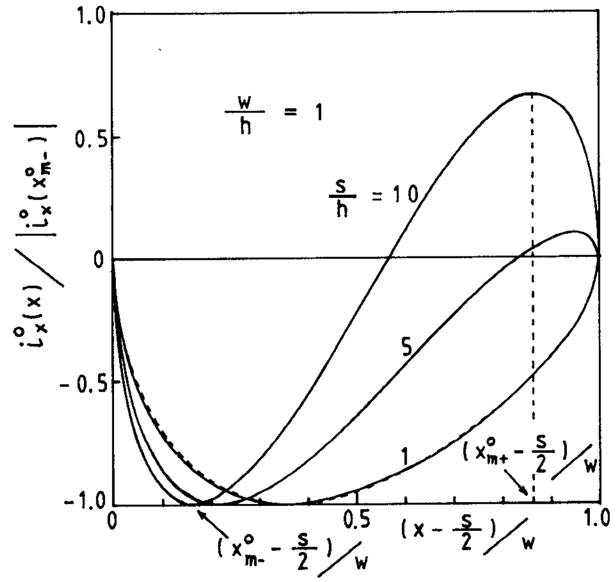

Fig. 9. Normalized transverse current distributions $i_x^o(x)/|i_x^o(x_{m-}^o)|$ on the strip for the cases of $w/h=1$, $s/h=1$, 5, 10, and $\epsilon^*=2$, 4, 8, 16 (odd mode). —— Present method. ------ Approximate formula [10].

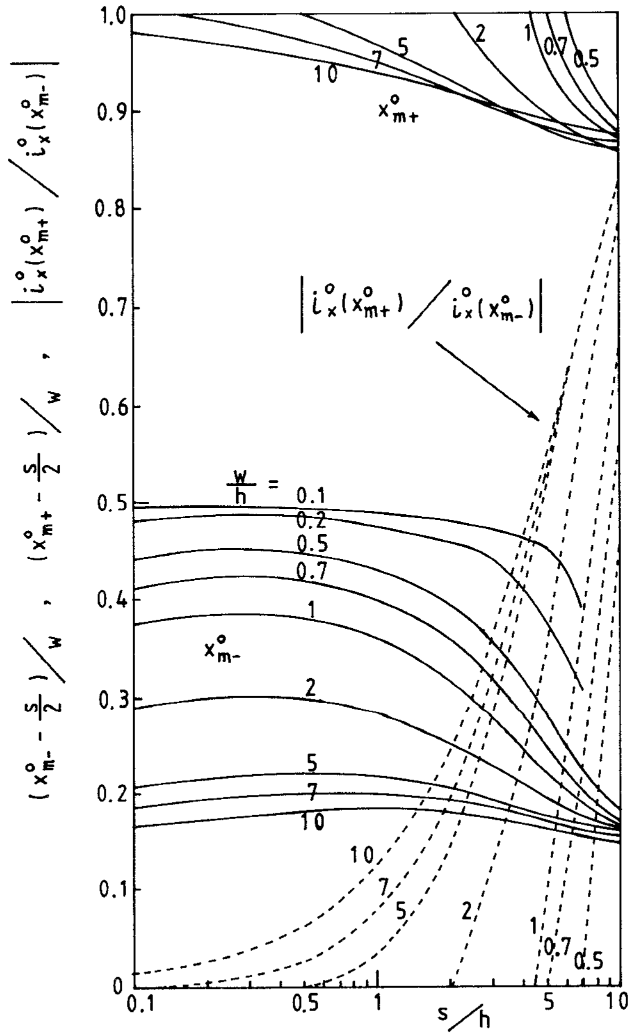

Fig. 10. The results for positive and negative extrema of transverse current distributions (odd mode).

mode, as s/h increases for $w/h = \text{constant}$, it begins to have a negative part in the vicinity of the inner edge when s/h is larger than some value, but for the odd mode it begins to have a positive part in the vicinity of the outer edge. These can be guessed by looking at the curves shown in Figs. 6 and 9 together with the results shown in Figs. 7 and 10. We cannot distinguish the curves for the cases of different ϵ^* in Figs. 4, 5, 6, 8, and 9, and therefore may conclude that the dependence of the normalized transverse current distribution on ϵ^* is extremely small, as for the single microstrip line. The curves shown in those figures satisfy the edge singularity [6] which requires that $i_x^P(x)$ behave like $|x - s/2|^{1/2}$ and $|x-(s/2+w)|^{1/2}$ near the inner edge and the outer edge of a strip, respectively.

V. Conclusions

The normalized longitudinal and transverse current distributions on coupled microstrip lines have been derived and their dependence on ϵ^*, w/h, and s/h has been explained. Accurate closed-form expressions for these distributions can be derived, although they are not given explicitly in this paper. Good agreement is shown between the theory and the approximate results. Using these current distributions, the dispersion characteristics of coupled microstrip lines will be investigated in the near future using spectral-domain analysis.

Acknowledgment

The authors would like to thank the reviewers and the editor for their valuable advice and helpful corrections to the grammar.

References

[1] T. Itoh and R. Mittra, "Spectral-domain approach for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496–499, July 1973.

[2] T. Itoh and R. Mittra, "A technique for computing dispersion characteristics of shielded microstriplines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 896–898, Oct. 1974.

[3] E. F. Kuester and D. C. Chang, "An appraisal of methods for computation of the dispersion characteristics of open microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 691–694, July 1979.

[4] E. J. Denlinger, "A frequency dependent solution for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30–39, Jan. 1971.

[5] M. K. Krage and G. I. Haddad, "Frequency-dependent characteristics of microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 678–688, Oct. 1972.

[6] R. Mittra and S. W. Lee, *Analytical Techniques* in the *Theory of Guided Waves*. New York: Macmillan, 1971, pp. 4–11.

[7] M. Kobayashi, "Analysis of the microstrip and the electrooptic light modulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 119–126, Feb. 1978.

[8] M. Kobayashi, "Longitudinal and transverse current distributions on microstriplines and their closed-form expression," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 784–788, Sept. 1985.

[9] M. Kobayashi and F. Ando, "Dispersion characteristics of open microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 101–105, Feb. 1987.

[10] M. Kobayashi and H. Momoi, to be submitted for publication.

Masanori Kobayashi (M'79) was born in Niigata, Japan, on June 17, 1947. He received the B.E. and M.E. degrees in electrical engineering from Ibaraki University, Ibaraki, Japan, in 1970 and 1972, respectively, and the D.E. degree in electrical and electronic engineering from the Tokyo Institute of Technology, Tokyo, Japan, in 1981.

Since April 1972 he has been with the Department of Electrical Engineering at Ibaraki University. He was a Research Assistant from April 1972 to March 1981 and a Lecturer from April 1981 to July 1982. Since August 1982, he has been an Associate Professor. His research interests are in the areas of microstrip transmission lines, dielectric optical waveguides, magnetic elements, and relativistic electromagnetic theory.

Dr. Kobayashi is a member of the Institute of Electrical Engineers of Japan and the Institute of Electronics, Information and Communication Engineers of Japan.

Hidetoshi Momoi was born in Fukushima, Japan, on June 19, 1962. He received the B.E. and M.E. degrees in electrical engineering from Ibaraki University, Ibaraki, Japan, in 1985 and 1987, respectively. In April 1987, he joined NTT, Tokyo, Japan.

Mr. Momoi is a member of the Institute of Electronics, Information and Communication Engineers of Japan.